



# Threshold production of the $\Theta^+$ in a polarized proton reaction

Seung-II Nam<sup>a,b</sup>, Atsushi Hosaka<sup>a</sup>, Hyun-Chul Kim<sup>b</sup>

<sup>a</sup> Research Center for Nuclear Physics (RCNP), Ibaraki, Osaka 567-0047, Japan

<sup>b</sup> Department of Physics and Nuclear Physics & Radiation Technology Institute (NuRI), Pusan National University, Busan 609-735, South Korea

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## Abstract

We compute the cross sections of the  $\Theta^+$  production near threshold for a polarized proton reaction,  $\vec{p}\vec{p} \rightarrow \Sigma^+\Theta^+$  which was recently proposed to determine unambiguously the parity of  $\Theta^+$ . Within theoretical uncertainties the cross sections for the allowed spin configuration are estimated; it is of order of one microbarn for the positive parity  $\Theta^+$  and about 1/10 microbarn for the negative parity  $\Theta^+$  in the threshold energy region, where the s-wave component dominates.

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The discovery of the  $\Theta^+$  [1–4] has triggered a tremendous amount of research activities both in theories and experiments [5]. Hadron physics has now experienced a new stage of development with unexpected richness. The surprise came not only with its relatively low mass but also with a very narrow width, though for the latter only the upper limit is known so far. This feature is also shared by the recently observed  $\Xi$  states [6]. Quantum numbers such as spin

and parity are not yet known neither. Since the parity reflects the internal dynamics of hadrons, it is crucially important to determine it by experiment and to understand by theory. The present theoretical situation, however, is not settled; chiral theories including the pioneering chiral soliton models [7–9], and the diquark model [10] predict positive parity, while recent lattice [11,12] and sum rule calculations [13,14] are on the other side.

Very recently, an unambiguous method was proposed in order to determine the parity of the  $\Theta^+$  by using the reaction [15]

$$\vec{p} + \vec{p} \rightarrow \Theta^+ + \Sigma^+ \quad \text{near threshold.} \quad (1)$$

E-mail addresses: [sinam@rcnp.osaka-u.ac.jp](mailto:sinam@rcnp.osaka-u.ac.jp) (S.-I. Nam), [hosaka@rcnp.osaka-u.ac.jp](mailto:hosaka@rcnp.osaka-u.ac.jp) (A. Hosaka), [hchkim@pusan.ac.kr](mailto:hchkim@pusan.ac.kr) (H.-C. Kim).

This reaction has been previously considered for the production of the  $\Theta^+$  [16], but it has turned out that it does more for the determination of the parity, in contrast with a number of recent attempts using other reactions which needed particular production mechanism [17]. In order to extract information of parity from (1), the only requirement is that the final state is dominated by the s-wave component. The s-wave dominance in the final state is then combined with the Fermi statistics of the initial two protons and conservations of the strong interaction, establishing the selection rule: *If the parity of  $\Theta^+$  is positive, the reaction (1) is allowed at the threshold region only when the two protons have the total spin  $S = 0$  and even values of relative momenta  $l$ , while, if it is negative the reaction is allowed only when they have  $S = 1$  and odd  $l$  values.* This situation is similar to what was used in determining the parity of the pion [18]. Experimentally, the pure  $S = 0$  state may not be easy to set up. However, an appropriate combination of spin polarized quantities allows to extract information of the  $S = 0$  state.

In this Letter, we perform calculations for production cross sections of (1). Our purposes are:

- (1) To check that the production reaction is indeed dominated by the s-wave (in other word, there is no accidental vanishing of s-wave contributions to invalidate the above selection rule);
- (2) To estimate production cross sections within the present knowledge of theoretical models.

In order to estimate the production rate, we calculate the Born diagrams of pseudoscalar kaon ( $K(498)$ ) and vector  $K^*$  ( $K^*(892)$ ) exchanges, which are minimally needed for the present reaction (Fig. 1). Assuming that the parity of the  $\Theta^+$  is positive, we can take effective interaction Lagrangian as follows:

$$\mathcal{L}_{KN\Theta} = i g_{KN\Theta} \bar{\Theta} \gamma_5 K N + (\text{h.c.}), \quad (2)$$

$$\mathcal{L}_{KN\Sigma} = i g_{KN\Sigma} \bar{\Sigma} \gamma_5 K N + (\text{h.c.}), \quad (3)$$

$$\begin{aligned} \mathcal{L}_{K^*N\Theta} = & -g_{K^*N\Theta} \bar{\Theta} \gamma^\mu K_\mu^* N \\ & + \frac{g_{K^*N\Theta}^T}{M_\Theta + M_N} \bar{\Theta} \sigma^{\mu\nu} \partial_\mu K_\nu^* N + (\text{h.c.}), \end{aligned} \quad (4)$$

$$\begin{aligned} \mathcal{L}_{K^*N\Sigma} = & -g_{K^*N\Sigma} \bar{\Sigma} \gamma^\mu K_\mu^* N \\ & + \frac{g_{K^*N\Sigma}^T}{M_\Sigma + M_N} \bar{\Sigma} \sigma^{\mu\nu} \partial_\mu K_\nu^* N + (\text{h.c.}) \end{aligned} \quad (5)$$

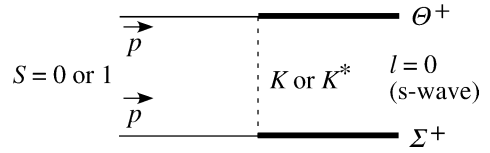


Fig. 1. Born diagrams for  $\vec{p}\vec{p} \rightarrow \Theta^+ \Sigma^+$ .

with standard notations. If the parity of  $\Theta^+$  is negative,  $\gamma_5$  matrix in (2) should be removed and in (4)  $\gamma_5$  should be inserted. For the coupling terms of  $\Sigma^+$ , we employ the values estimated from the previous analysis;  $g_{KN\Sigma} = 3.54$ ,  $g_{K^*N\Sigma} = -2.46$  and  $g_{K^*N\Sigma}^T = 1.15$  [19]. Since the couplings to the  $\Theta^+$  is not known, we investigate several cases with different parameter values. For  $g_{KN\Theta}$  we mostly employ  $g_{KN\Theta} = 3.78$ , which is fixed by  $\Gamma_{\Theta^+ \rightarrow KN} = 15$  MeV. For each case, we employ for the unknown vector  $K^*$  couplings,  $|g_{K^*N\Theta}| = |g_{KN\Theta}|/2$ , as suggested by Ref. [20]. The tensor couplings are then varied within  $|g_{K^*N\Theta}^T| \leq 2|g_{K^*N\Theta}| = |g_{KN\Theta}|$  in order to see the model dependence of this process. As for the form factor, we employ the following form of the monopole type:

$$F(q^2) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2}, \quad (6)$$

where  $q^2$  is the four momentum square and  $m$  the mass of the exchanged particle (either  $K$  or  $K^*$ ). The cut-off parameter  $\Lambda$  is chosen to be  $\Lambda = 1$  GeV. In Ref. [21] the authors employed a different type of form factor. However, the monopole type is more often used for meson–baryon vertices. In any event, the main points in the following discussions will not be changed by the use of different form factors.

The calculation of the scattering amplitude is straightforward once having the interaction, Eqs. (2)–(5). In Fig. 2, total cross sections near threshold region are shown as functions of the energy in the center of mass system  $\sqrt{s}$  ( $\sqrt{s}_{\text{th}} = 2729.4$  MeV). The left (right) panel is for the positive (negative) parity  $\Theta^+$ , where the allowed initial state has  $S = 0$  and even  $l$  ( $S = 1$  and odd  $l$ ). For the allowed channels, five curves are shown using different coupling constants of  $g_{K^*N\Theta}$  and  $g_{K^*N\Theta}^T$ ; zero and four different combinations of signs with the absolute values  $|g_{K^*N\Theta}| = 2|g_{K^*N\Theta}| = |g_{KN\Theta}|$ , as indicated by the pair of labels in the figures, ( $\text{sgn}(g_{K^*N\Theta})$ ,  $\text{sgn}(g_{K^*N\Theta}^T)$ ). As shown in the figure, cross sections vary with about

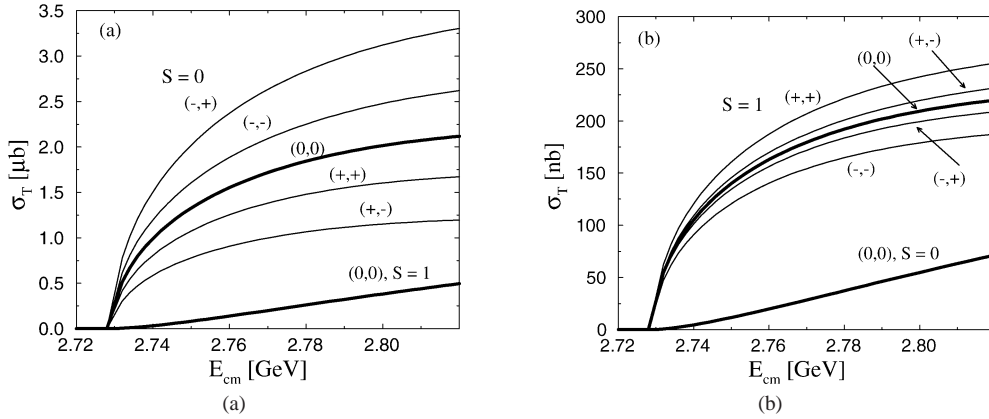


Fig. 2. Total cross sections near the threshold: (a) for positive parity  $\Theta^+$  where the allowed channel is ( $S = 0$ , even  $l$ ) and (b) for negative parity  $\Theta^+$  where the allowed channel is ( $S = 1$ , odd  $l$ ). The labels  $(+,+)$  etc. denote the signs of  $g_{K^*N\Theta}$  and  $g_{K^*N\Theta}^T$  relative to  $g_{KN\Theta}$ . The solid lines in the bottom is the cross sections for the forbidden channels.

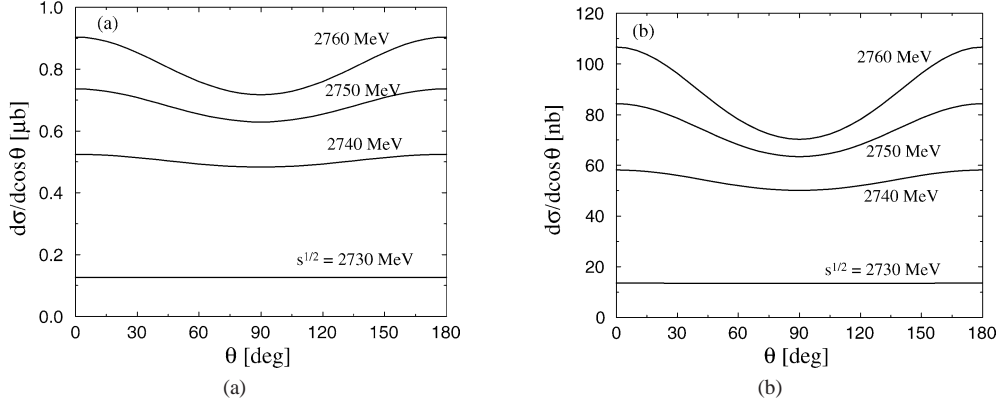


Fig. 3. Angular dependence of the production cross sections near the threshold in the center of mass frame: (a) for positive parity  $\Theta^+$  and (b) for negative parity  $\Theta^+$ . The labels denote the total incident energy  $\sqrt{s}$ .

50% from the mean value for the vanishing  $K^*$  exchanges. For the forbidden channels only the case of vanishing  $K^*N\Theta$  coupling constants is shown; cross sections using finite coupling constants vary within about 50% just as for the allowed channels. In both figures, the s-wave threshold behavior is seen for the allowed channels as proportional to  $(s - s_{\text{th}})^{1/2}$ , while the forbidden channels exhibit the p-wave dependence of  $(s - s_{\text{th}})^{3/2}$  and with much smaller values than the allowed channel. The suppression factor is given roughly by  $[(\text{wave number}) \cdot (\text{interaction range})]^2 \sim k/m_K \sim 0.1$  ( $k = \sqrt{2m_K E}$ ), as consistent with the results shown in the figures.

From these results, we conclude that the absolute value of the total cross section is of the order 1 [ $\mu\text{b}$ ]

for the positive parity  $\Theta^+$  and of the order 0.1 [ $\mu\text{b}$ ] for the negative parity  $\Theta^+$ . The fact that the positive parity case has larger cross section is similar to what was observed in the photoproduction and hadron induced reaction also [17]. This is due to the p-wave nature of the  $KN\Theta$  coupling with a relatively large momentum transfer for the  $\Theta^+$  production. When the smaller decay width of  $\Theta^+$  is used, the result simply scales as proportional to the width, if the  $K^*N\Theta$  couplings are scaled similarly.

In Fig. 3, we show the angular dependence of the cross section in the center of mass system for several different energies above the threshold,  $\sqrt{s} = 2730, 2740, 2750$  and  $2760$  MeV. Here, only  $K$  exchange is included but without  $K^*$  exchanges. The

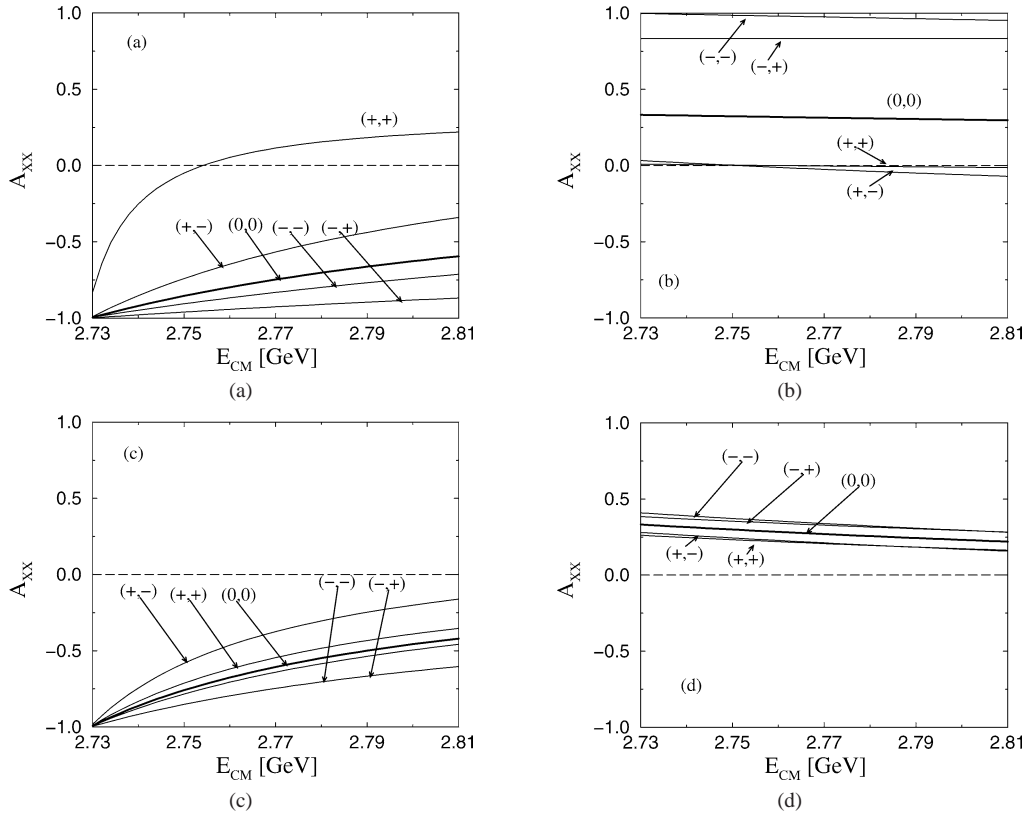


Fig. 4.  $A_{xx}$  for the positive (a) and negative (b) parities are drawn without the form factor. As for the cases with the form factor, we also show it for the positive (c) and negative (d) ones.

angular dependence with  $K^*$  exchanges included is similar but with absolute values scaled as in the total cross sections. Thus, we can verify again that the s-wave dominates the production reaction up to  $\sqrt{s} \lesssim 2750$  MeV.

Recently, in Ref. [22], the authors discussed the experimental methods and observables to determine the parity of the  $\Theta^+$  baryon with the polarized proton beam and target. They discussed the spin correlation parameter  $A_{xx}$  as well as cross sections. It is computed by

$$A_{xx} = \frac{(^3\sigma_0 + ^3\sigma_1)}{2\sigma_0} - 1, \quad (7)$$

where  $\sigma_0$  is the unpolarized total cross sections and the polarized cross section are denoted as  $^{2S+1}\sigma_{S_z}$ . In Fig. 4 we present  $A_{xx}$  including  $K^*$  exchange with and without the form factors. As shown in the figures  $A_{xx}$  reflects very clearly the differences of the parity

of  $\Theta^+$ . When the form factor is included, the five cases of different  $K^*$  coupling constants are similar and the resulting  $A_{xx}$  fall into well the region as indicated in Ref. [22]. If the form factor is not included, there is an accidental cancellation in the allowed s-wave amplitude for the  $(+, +)$  case. Hence, the p-wave contribution becomes significant at relatively low energy, which changes the sign of  $A_{xx}$  at  $E_{CM} \sim 2.75$  GeV in the case of positive parity (Fig. 4(a)). However, the sign of  $A_{xx}$  is one in the vicinity of the threshold region, as expected in the selection rule.

In an actual experiment, it is necessary to detect the  $\Sigma$  also at the threshold region. Due to small energy (or velocity) of the final-state particles in the center of mass system, produced  $\Sigma$  must be found inside a very narrow cone forward peaked in the laboratory frame. Because of this fact, measurement at the existing facility of a fixed target, such as COSY, would require an experimental challenge.

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